Čerenkov radiation by neutrinos in a supernova core

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Abstract

Neutrinos with a magnetic dipole moment propagating in a medium with a velocity larger than the phase velocity of light emit photons by the Čerenkov process. The Čerenkov radiation is a helicity flip process via which a left-handed neutrino in a supernova core may change into a sterile right-handed one and free-stream out of the core. Assuming that the luminosity of such sterile right-handed neutrinos is less than 10^{53} ergs/sec gives an upper bound on the neutrino magnetic dipole moment $\mu_\nu < 0.2 \times 10^{-13} \mu_B$. This is two orders of magnitude more stringent than the previously established bounds on μ_ν from considerations of supernova cooling rate by right-handed neutrinos.

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The observation of neutrinos [1] from the supernova SN1987A has provided a number of constraints on the properties of neutrinos [2–4]. Most of the mechanisms for the constraints on the mass and magnetic moment of the neutrinos depend upon the the helicity flip of a left-handed neutrino into a sterile right-handed one, which can free-stream out of the core and hence deplete the energy of the supernova core within a timescale of ~ 1 sec. Since the observed time-scale of neutrino emission [1] is of the order of 1-10 secs, it is expected that the luminosity of the right-handed neutrinos is less than 10^{53} ergs/ sec, which is the total neutrino luminosity of the supernova. The mechanism for helicity flip caused by a neutrino magnetic moment that have been considered so far are (i) helicity flip in an external magnetic field of the neutron star in the supernova core [2] and (ii) helicity flip by scattering with charged fermions *i.e.* the processes $\nu_L e^- \to \nu_R e^-$, $\nu_L p \to \nu_R p$ [3]. The process (i) leads to an upper bound $\mu_\nu < 10^{-14}\mu_B$ ($\mu_B = e/2m_e$, the Bohr magneton), but is unreliable since it relies on a high magnetic field ($\sim 10^{14}$ Gauss) in a supernova core which has not been observed. The scattering process (ii) leads to an upper bound $\mu_\nu < (0.2 - 0.8) \times 10^{-11}\mu_B$ [3].

In this letter we propose a third mechanism for the neutrino helicity flip which occurs via a Čerenkov radiation process in the medium of the supernova core. In the supernova core the refractive index of photons is determined by the electric permitivity of the e^-, e^+ plasma and the paramagnetic susceptibility of the non-relativistic, degenerate neutron and proton gas. We find that Čerenkov emission of a photon from a neutrino is allowed in the photon frequency range $\omega_p \mu^{1/2}/(\mu - 1)^{1/2} \le \omega \le 2E(\mu^{1/2} - 1)/(\mu - 1)$, (where ω_p is the plasma frequency, μ is the magnetic permeability, and E is the initial neutrino energy). Since the Čerenkov emission process is due to the magnetic dipole operator $\mu_\nu \sigma_{\mu\nu} k_\nu \epsilon_\mu$, it is a helicity flipping process $\nu_L \to \nu_R \gamma$. The helicity flipping is more efficient in the Čerenkov process because unlike the process (i) there is no dependence on external magnetic field and unlike (ii) it is a single vertex process, so the rate is larger than the scattering rate $\nu_L e^- \to \nu_R e^-$ by $(\alpha_{em} e^{-\tilde{\mu}_e/T})^{-1}$, where the exponential factor is due to the Pauli blocking of the outgoing charged fermion. We compute the luminosity Q_{ν_R} of the right-handed neutrinos produced

by the Čerenkov process. The constraint that $Q_{\nu_R} < 10^{53}$ ergs/sec (the total observed luminosity) leads to the bound on the neutrino magnetic moment $\mu_\nu < 0.2 \times 10^{-13} \mu_B$. This is two order of magnitude improvement over the previously established bound [3] owing to the fact that the Čerenkov radiation is a single vertex process unlike the scattering processes considered in ref. [3].

The amplitude for the Čerenkov radiation process $\nu_L(p) \to \nu_R(p')\gamma(k)$ is given by

$$\mathcal{M} = \frac{\mu_\nu}{n} \overline{u}(p', s') \sigma^{\mu\nu} k_\nu u(p, s) \epsilon_\mu(k, \lambda), \tag{1}$$

where μ_ν is the magnetic dipole moment of neutrino and n is the refractive index of the medium. The transition rate of the Čerenkov process is given by

$$\Gamma = \frac{1}{2E} \int \frac{d^3p'}{(2\pi)^3 2E'} \frac{d^3k}{(2\pi)^3 2\omega} (2\pi)^4 \, \delta^{(4)}(p - p' - k) |\mathcal{M}|^2, \tag{2}$$

where $p = (E, \mathbf{p})$, $p' = (E', \mathbf{p}')$ and $k = (\omega, \mathbf{k})$ are the four momenta of the incoming neutrino, outgoing neutrino and the emitted photon respectively. Using the identity

$$\int \frac{d^3p'}{2E'} = \int d^4p' \;\, \theta(E') \, \delta(p'^2 - m_\nu^2),$$

and integrating over the δ function in (2) we obtain

$$\Gamma = \frac{1}{16\pi} \int \frac{k^2 dk}{E^2 \omega^2 n} d(\cos\theta) \, \delta\left(\frac{2\omega E - k^2}{2|\mathbf{k}||\mathbf{p}|} - \cos\theta\right) |\mathcal{M}|^2, \tag{3}$$

where θ is the angle between the emitted photon and the incoming neutrino.

In a medium with the refractive index $n (= |\mathbf{k}|/\omega)$, the δ function in (3) constrains $\cos\theta$ to have the value

$$\cos\theta = \frac{1}{nv} [1 + \frac{(n^2 - 1)\omega}{2E}], \tag{4}$$

where $v = |\mathbf{p}|/E$ is the particle velocity and $k^2 = -(n^2 - 1)\omega^2$. It is clear that the kinematically allowed region for the Čerenkov process is where $|\cos\theta|$ given by (4) is ≤ 1.

Evaluating $|\mathcal{M}|^2$ from (1) and substituting it in (3) and performing the integral over δ function and using (4) for $\cos\theta$, we have the expression for transition rate for the Čerenkov process [9–11]

$$\Gamma = \frac{\mu_\nu^2}{16\pi E^2}\int_{\omega_1}^{\omega_2} d\omega \frac{(n^2-1)^2}{n^2}[\{4E^2 + 4m_\nu^2 \frac{n^2}{(n^2-1)}\}\omega^2 - 4E\omega^3 - (n^2-1)\omega^4], \tag{5}$$

where the limits of the integral give the range of frequency allowed for the Čerenkov photon and the refractive index n is, in general, a function of ω.

The refractive index of photons in a medium can be expressed as $n^2 = \epsilon\mu$, where ϵ and μ are the electric permitivity and magnetic permiability of the medium. In the supernova core, the medium is a plasma consisting of degenerate electrons, protons and neutrons at temperature $T \approx 60$ MeV [3,5]. The permitivity ϵ is given by

$$\epsilon = (1 - \frac{\omega_p^2}{\omega^2}), \tag{6}$$

where $\omega_p = (4\alpha_{em}/3\pi)^{1/2}\tilde{\mu}_e$ [6] is the plasma frequency that is determined by the chemical potential of the electrons $\tilde{\mu}_e$. In a non-magnetic plasma where magnetic permitivity $\mu = 1$, the refractive index $n = (1-\omega_p^2/\omega^2)^{1/2} < 1$ and the Černkov process is therefore forbidden. In the supernova core, however there is a large density of non-relativistic neutrons and protons which contribute to the paramagnetic susceptibility χ, (related to the magnetic permiability $\mu = 1 + 4\pi\chi$) through their magnetic dipole moments.

Treating the neutrons and protons in the supernova core as degenerate Fermi gas, the magnetic susceptibility becomes independent of temperature and is given as a function of the photon wavelenght k as [7,8]

$$\chi_i(k) = \frac{1}{2\pi^2}(2m_i)^{3/2}{\mu_i}^2\tilde{\mu}_i^{1/2}(\frac{1}{2})(1 + \frac{4{k_{fi}}^2 - k^2}{4k_{fi}\ k}\ ln|\frac{2k_{fi}+k}{2k_{fi}-k}|\ \ ), \tag{7}$$

where m_i is the mass, μ_i is the magnetic moment, $\tilde{\mu}_i$ is the chemical potential and $k_{fi} = (2m_i\tilde{\mu}_i)^{1/2}$ is the fermi momentum of the $i\ th$ fermion species. In the supernova core the photon wavelenghts $|k| \sim T << k_{fi}$ as $T \sim 60Mev$ and the fermi momentum $k_{fn} \sim 894MeV$ for neutrons (with chemical potential $\tilde{\mu}_n \sim 400MeV$) and $k_{fp} \sim 748MeV$ for protons (with $\tilde{\mu}_p \sim 280MeV$) [3,4]. The wavenumber dependence of the paramagnetic susceptibility (7) can be expanded as a series in the small parameter (k/k_f) as

$$\chi_i(k) = \frac{1}{2\pi^2}(2m_i)^{3/2}{\mu_i}^2\tilde{\mu}_i^{1/2}(1 - (\frac{k}{2k_{fi}})^2 + \cdots), \tag{8}$$

The contribution of the wavenumber dependent terms is about 10^{-3} smaller compared to the leading order term and will be neglected in the present analysis. Only the non-relativistic fermions (neutrons and protons) contribute to the dipole susceptibility [7]. If the photon frequency is close to the nucleon mass, such that the photon could produce significant numbers of nucleon pairs during propagation then the expression (7) for the diamagnetic susceptibity will no longer be valid. Therefore our conclusions are subject to the assumption that the photon freqencies (which we assume to be close to the temperature $T \sim 60MeV$) are small compared to the nucleon masses of $\sim 1GeV$. In our treatment we do not consider the corrections to (7) due to pair production [8] at frequencies close to the nucleon masses. Taking the chemical potentials $\tilde{\mu_p} \approx 280$ MeV, $\tilde{\mu_n} \approx 400$ MeV [3,4] for the protons and neutrons respectively, the susceptibility $\chi = \chi_p + \chi_n = 0.15$ and the magnetic permiability turns out to be $\mu = 2.88$. The electrons being relativistic ($m << T$), do not contribute to the magnetic susceptibility. The electrons with chemical potential $\tilde{\mu_e} \approx 280$ MeV and plasma frequency ω_p contribute to the refractive index via the electric permitivity ϵ. Consequently, the refractive index of the non-relativistic degenerate neutrons and protons and relativistic electrons in a supernova core is given by

$$n(\omega) = (\mu\epsilon)^{1/2} = [1 + 4\pi(\chi_n + \chi_p)]^{1/2}(1 - \frac{\omega_p^2}{\omega^2})^{1/2}. \tag{9}$$

By combining the kinematic constraint (4) with the above expression for the refractive index (9), we find that Čerenkov radiation by neutrinos is kinematically allowed for the range of the frequency ω given by

$$\frac{\omega_p \mu^{1/2}}{(\mu - 1)^{1/2}} \leq \omega \leq \frac{2E(\mu^{1/2} - 1)}{(\mu - 1)}, \tag{10}$$

where we have taken $|p| = E(1 - m^2/E^2)^{1/2} \simeq E$ since we are dealing with extremely relativistic neutrinos with $m^2/E^2 < 10^{-12}$.

Keeping terms upto second order in ω in the expression for $n(\omega)$, and neglecting the second term (since $m_\nu \sim 1$ eV and $\omega \sim 60$ MeV) in the expression for Γ given in (5), the transition rate for the Čerenkov process in the supernova core is evaluated to be

$$
\begin{aligned}
\Gamma &= \frac{\mu_\nu^2}{16\pi E^2}\frac{(\mu-1)}{\mu}\int_{\omega_1}^{\omega_2} d\omega[4E(\omega-E)\{(1-\mu)\omega^2+\omega_p^2(\mu+1)\} \\
&\quad +(\mu-1)\omega^2\{(1-\mu)\omega^2+\omega_p^2(2\mu+1)\}] \\
&= \frac{\mu_\nu^2 E}{6\pi\mu^2}[\frac{2}{5}E^2\frac{(\mu^{1/2}-1)^2(4\mu^{3/2}+16\mu-15)}{(1+\mu^{1/2})^2}+\omega_p^2\mu(1-\mu^{3/2})]. \qquad (11)
\end{aligned}
$$

The Čerenkov process $\nu_L(p) \rightarrow \nu_R(p')\gamma(k)$ changes the ν_L's to sterile ν_R's which can free Stream out of the supernova core. The luminosity of the sterile ν_R's is the product of the energy taken by each right-handed particle *i.e.* $(E-\omega)$ and the total number of right-handed particles produced per unit volume as given by (11), multiplied with the volume of the supernova core and is found to be

$$
\begin{aligned}
Q_{\nu_R} &= \frac{3V\mu_\nu^2}{16\pi}\frac{(\mu-1)}{\mu}\int_0^\infty \frac{dE}{E^2}[f_\nu(E)-f_{\overline{\nu}}(E)]E^2\int_{\omega_1}^{\omega_2} d\omega(E-\omega)\times \\
&\quad [4E(\omega-E)\{(1-\mu)\omega^2+\omega_p^2(\mu+1)\}+(\mu-1)\omega^2\{(1-\mu)\omega^2+\omega_p^2(2\mu+1)\}], \qquad (12)
\end{aligned}
$$

where $f_\nu(E)=[e^{(E-\tilde{\mu_\nu})/T}+1]^{-1}$ and $f_{\overline{\nu}}(E)=[e^{(E+\tilde{\mu_\nu})/T}+1]^{-1}$ are the statistical distribution function of the ν_L and $\overline{\nu_L}$ in the supernova core, $\tilde{\mu_\nu}$ is the chemical potential of the neutrino and the factor of 3 is due to the contributions from all three neutrino flavours as the cooling proceeds through the emission of $\nu\overline{\nu}$ pairs of all flavours, created in thermal equilibrium [12]. Performing the integrals over E, the luminosity of right handed neutrinos is obtained as

$$
\begin{aligned}
Q_{\nu_R} &= \frac{V\mu_\nu^2\tilde{\mu_\nu}(\mu^{1/2}-1)}{210\pi\mu(1+\mu^{1/2})^3}[16\tilde{\mu_\nu}^2(3\tilde{\mu_\nu}^4+21\pi^2T^2\tilde{\mu_\nu}^2+49\pi^4T^4)\mu(\mu^{1/2}-1) \\
&\quad -7\omega_p^2(3\tilde{\mu_\nu}^4+10\pi^2T^2\tilde{\mu_\nu}^2+7\pi^4T^4)(1+4\mu^{1/2}+7\mu+10\mu^{3/2}+8\mu^2+2\mu^{5/2})]. \qquad (13)
\end{aligned}
$$

We take the volume $V \approx 4\times10^{18}$ cm^3, $\tilde{\mu_\nu} \approx 160$ MeV, $T \approx 60$ MeV [3] for the supernova core parameters within 1 second after collapse. Using these numbers we obtain the luminosity to be

$$
Q_{\nu_R} = 0.98\times10^{53} \quad \mu_\nu^2 \quad GeV^4, \qquad (14)
$$

in terms of the magnetic moment of neutrino μ_ν. Assuming that the entire energy of the core collapse is not carried out by the right handed sterile neutrinos, *i.e.* $Q_{\nu_R} < 10^{53}$ ergs/sec, we have from (14) the upper bound on the neutrino magnetic dipole moment given by

$$\mu_\nu < 0.2 \times 10^{-13} \mu_B. \tag{15}$$

Varying the core temperature of the supernova in the range $30 - 70$ MeV, the upper bound (15) is seen to fall in the range $(0.59 - 0.15) \times 10^{-13} \mu_B$ respectively. The upper bound on neutrino magnetic moment given in (15) is two orders of magnitude better than the previously established [3] upper bound from the ν_R luminosity of supernova. The process for generating ν_R in the supernova core considered in ref. [3] is via the helicity flip scattering $\nu_L e^- \to e^- \nu_R$ and $\nu_L p^- \to p^- \nu_R$ etc. This process has an extra electromagnetic vertex and a Pauli blocking factor for the outgoing charged fermion compared to the process that we have considered and is suppressed compared to the process considered here by the factor $\alpha_{em}\, e^{-\tilde{\mu}_e/T}$. That accounts for the more stringent bound we have compared to ref. [3].

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